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BATTERWOT'S CHARACTERISTIC AND INDICES OF A ROCKET STABILIZING SYSTEM

Встановлено зв'язок між радіусом півкола для розміщення коренів характе-ристичного поліному, коефіцієнтами закону регулювання, статичною похибкою і приведеною роботою виконавчого пристрою протягом перехідного процесу ком-пенсації постійного збурювального прискорення. Результати можуть бути викори-стані при розробці системи як альтернативний варіант для прийняття технічних рішень.

Ключові слова: *модальне управління, закон регулювання, статична похибка.*

Установлена связь между радиусом полуокружности для расположения корней характеристического полинома, коэффициентами закона регулирования, статической ошибкой и приведенной работой исполнительного устройства в течении переходного процесса компенсации постоянного возмущающего ускорения. Результаты могут быть использованы при разработке системы как альтернативный вариант для принятия технических решений.

Ключевые слова: *модальное управление, закон регулирования, статическая ошибка.*

The connection between radius of a semicircle for placement of the characteristic polynomial's roots, coefficients of a law of control, static error and reduced work during the transient process for compensation of a constant disturbing acceleration is estab-lished. The results can be used during system elaboration as an alternative option for engineering solution.

Keywords: *modal control, law of control, static error.*

Introduction. One of the options for determining the law of regulation in the system of stabilization (SS) of the rocket movement is to use the modal control method, which provides a given root location of the characteristic polynomial (CP) and the corresponding transient function. The harmonization of the requirements for the margin of SS stability, the quality of the transition process, the accuracy and the requirements for the power of the actuator requires the design of compromise decisions. Placing the roots of the CP according to Butterworth evenly over a semicircle of a certain radius is appropriate in terms of the frequency response of the SS in the low frequency range, but it remains open to question the accuracy of the compensation of perturbations and energy costs in transients.

The aim of the work is to establish a connection between the Butterworth parameter – the radius of the semicircle on the plane of the root of the CP, the quantitative estimation of accuracy, and the work of the actuator in the transient process of compensation of constant perturbation acceleration.

The work materials extend the methodological basis for the design of SS missiles and can be used as one of the options for determining the law of regulation based on the coefficients of perturbation equations around a certain point of the trajectory.

Refereces review. The main results obtained for the design of the SS as an integral part of the missile or spacecraft motion control system and the choice of regulation law (RL) are contained in a monograph [1]. Requirements for the frequency response of the actuator (RA) are set taking into account the oscillators due to the ultimate rigidity of the housing and the liquid fuel in the tanks.

Fluid fluctuations in tanks or payload structures significantly complicate the stabilization of movement and require special measures in the design of the project, in particular, rational choice of tank shape and installation of dampers [2].

When the mathematical model of the SS takes into account the fluctuations of the liquid fuel and the ultimate rigidity of the housing, as a rule, the dimension of the state vector is greater than the number of additives RL, which makes it impossible to directly use the well-known method of analytical design of regulators, where coefficients on the quality criterion of the transition process. But the necessary margin of stability can be ensured provided that there is sufficient damping in the oscillators and the proper selection of the frequency response of the controller [3].

A technique for optimization of RL by the criterion of "stability probability" is developed, since the parameters of the rocket are known with a certain error [4]. Its effectiveness is confirmed by the eighth-order model, which, in addition to rotational, takes into account the motion of the center of mass, the elasticity of the housing and the dynamic characteristics of the RA.

One of the methods for calculating linear RL coefficients, in which the quantitative estimation (criterion) of the quality of the transient SS perturbation compensation process becomes the least, is the use of a system of nonlinear Riccati equations. In [5], this system is presented for the fourth-order SS model, and in [6], for its second-order model, its analytical solution is obtained and an algorithm for determining elements of a symmetric criterion matrix, which provides the specified SS parameters, in particular, global stability is provided.

Taking into account the given margin of stability by analytical methods dependences of indicators characterizing the error of compensation of linear in time of perturbation acceleration from coefficients of equations of motion, parameters of executive and correction devices are determined [7]. The results are obtained for the model of the SS of flat rotary motion without taking into account the perturbations of the center of mass, in which, apart from the traditional ones, there are additives proportional to the angular acceleration of the housing and the angular velocity of rotation of the equivalent steering body. These components make it possible to increase the stability region on the plane of the other two coefficients of the RL and to reduce the individual coordinates of the error vector, but the level of the high-frequency interference component increases in the signal of the controller.

The estimations of the accuracy of compensation of linear perturbation acceleration in the form of simple analytical dependences on the parameters of the rocket and the coefficients of RL were obtained [8]. The possibility of a compromise reconciliation of conflicting requirements for accuracy and margin of stability is shown. The perturbation of the motion of the center of mass and the analysis of the requirements for the power of the RA at transient processes of compensation of perturbations are not taken into account.

The analysis shows that the available sources do not sufficiently cover SS indicators, the RL of which was chosen by the modal control method, in particular, using standard Butterworth forms [9].

Materials and methods. At the initial stage of SS development, the oscillators of the elastic oscillations of the rocket body and the liquid fuel, as well as the inertia of the RA, are not taken into account. Then in the vicinity of a certain point of the trajectory the perturbed motion in the yaw plane is described by the equations:

$$\ddot{\psi} = a_{\psi\psi} \cdot \psi + a_{\psi\delta} \cdot \delta + m_y, \dot{V}_z = a_{z\psi} \cdot \psi + a_{z\delta} \cdot \delta + f_z, \delta = k_\psi \cdot \psi + k_\psi^{'} \cdot \dot{\psi} - k_z^{'} \cdot V_z, \quad (1)$$

where ψ, δ – angles of yaw and turn of the equivalent steering body, V_z – projection of the velocity of the center of mass of the rocket on the axis perpendicular to the plane of the trajectory; $a_{\psi\psi}, a_{\psi\delta}, a_{z\psi}, a_{z\delta}$ – traditional [1] designations of coefficients that depend on the parameters of the rocket and trajectory; f_z, m_y – perturbing linear and rotational acceleration; $k_\psi, k_\psi^{'}, k_z^{'}$ – RL coefficients.

To assess the accuracy of the SS equations (1) are written in matrix form:

$$\dot{x} = a \cdot x + f, \quad a = \begin{bmatrix} 0 & 1 & 0 \\ a_{\psi\psi} + a_{\psi\delta} \cdot k_\psi & a_{\psi\delta} \cdot k_\psi^{'} & -a_{\psi\delta} \cdot k_z^{'} \\ a_{z\psi} + a_{z\delta} \cdot k_\psi & a_{z\delta} \cdot k_\psi^{'} & -a_{z\delta} \cdot k_z^{'} \end{bmatrix}, \quad x = \begin{bmatrix} \psi \\ \dot{\psi} \\ V_z \end{bmatrix}, \quad f = c \cdot w = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} m_y \\ f_z \end{bmatrix} \quad (2)$$

According to the third-order Batterwot form [9], the roots $Q(s) = s^3 + \sum_{i=0}^{2} q_i \cdot s^i$ of the CP are located in the left half of the plane of the complex variable s in a semicircle with a radius ω_0, and the corresponding coefficients are the following ($j^2 = -1$):

$$\omega_0 \cdot \begin{bmatrix} -1 \\ -0.5 \pm j \cdot \sqrt{3}/2 \end{bmatrix}, q_0 = \omega_0^3, q_1 = 2\omega_0^2, q_2 = 2\omega_0. \quad (3)$$

The dependence of the RL on the Batterwot parameter is determined by equating the CP coefficients determined from equations (1) to the corresponding values (3):

$$k_\psi = \frac{2\omega_0^2 + a_{\psi\psi}}{|a_{\psi\delta}|}, \quad k_\psi^{'} = \frac{\omega_0 \cdot (2\Delta - a_{z\delta} \cdot \omega_0^2)}{\Delta \cdot |a_{\psi\delta}|}, \quad k_z^{'} = \frac{\omega_0^3}{\Delta}, \quad \Delta = a_{\psi\delta} \cdot a_{z\psi} - a_{z\delta} \cdot a_{\psi\psi}. \quad (4)$$

For the data Table 1 the coefficients of RL (4) are shown in Fig. 1. Their upper limit is absent unless one considers the power and performance requirements of the RA and remains within model (1).

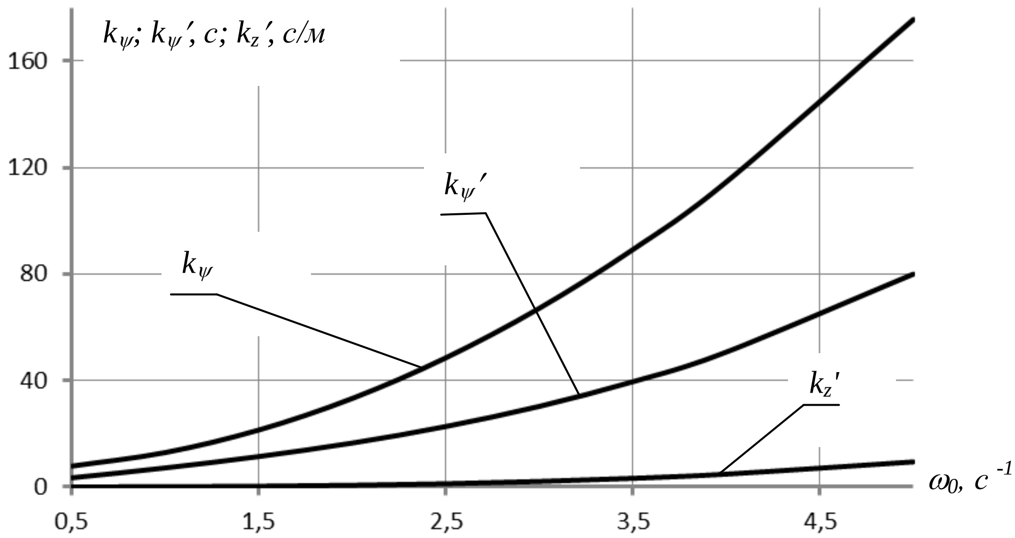

Fig.1 RL coefficients depending on Batterwot's parameter

Table 1

An example of the equations coefficients of the perturbed rocket motion

$a_{z\psi}$	$a_{z\delta}$	$a_{\psi\delta}$	$a_{\psi\psi}$
м/с^2		1/с^2	
-36.09	-1.441	-0.295	1.8113

The accuracy of SS under the action of constant perturbation acceleration is quantified by the error matrix

$$er_0 = -a^{-1} \cdot c; \tag{5}$$

where a, c were defined in (2). Consider the case in which the rotational perturbation acceleration m_y is caused by the projection of aerodynamic force on an z axis, which is applied at the center of aerodynamic pressure x_d . Then

$$m_y = k_i \cdot f_{z0}, \; k_i = \frac{m \cdot (x_\partial - x_t)}{I},$$

where m, I is the mass and moment of inertia of the rocket relative to the transverse axis passing through the center of mass with the coordinate x_t ; in this approach, the matrix (5) is transformed into an error vector.

Elemental transformations make it possible to obtain it analytically:

$$er0 = \frac{1}{k_z' \cdot \Delta} \cdot \begin{bmatrix} k_z' \cdot (a_{z\delta} \cdot k_i - a_{\psi\delta}) \\ 0 \\ k_i \cdot (a_{z\psi} + a_{z\delta} \cdot k_\psi) - a_{\psi\psi} - a_{\psi\delta} \cdot k_\psi \end{bmatrix}. \qquad (6)$$

Under the action of constant perturbation f_{z0} after the transition process, the vector of the SS state approaches $x_f = er0 \cdot f_{z0}$. This is confirmed by the numerical solution of the system of equations (2).

The analysis shows that the Batterwot parameter ω_0 affects only the third coordinate (Fig. 2) of the error vector (6), which depends on the value V_z after the transition process:

$$er0_1 = \frac{a_{z\delta} \cdot k_i - a_{\psi\delta}}{a_{\psi\delta} \cdot a_{z\psi} - a_{\psi\psi} \cdot a_{z\delta}}, \quad er0_2 = 0, \quad er0_3 = \frac{a_2 \cdot \omega_0^2 + a_0}{\omega_0^3},$$

$$a_2 = \frac{2 \cdot (k_i \cdot a_{z\delta} - a_{\psi\delta})}{|a_{\psi\delta}|}, \quad a_0 = \frac{a_{\psi\psi} \cdot (k_i \cdot a_{z\delta} - a_{\psi\delta})}{|a_{\psi\delta}|} + k_i \cdot a_{z\psi} - a_{\psi\psi}. \qquad (7)$$

One way to quantify the quality of the transition process is to determine the reduced work of the RA on the transient compensation process of constant perturbation acceleration at zero initial conditions [5].

Working for an interval of time dt is obviously a product of the torque created by the RA, the angle of rotation of the steering body, that is $M \cdot \dot{\delta} \cdot dt$ and during the transition process T it is equal to

$$A = \int_0^T \left| M(t) \cdot \dot{\delta}(t) \right| \cdot dt. \qquad (8)$$

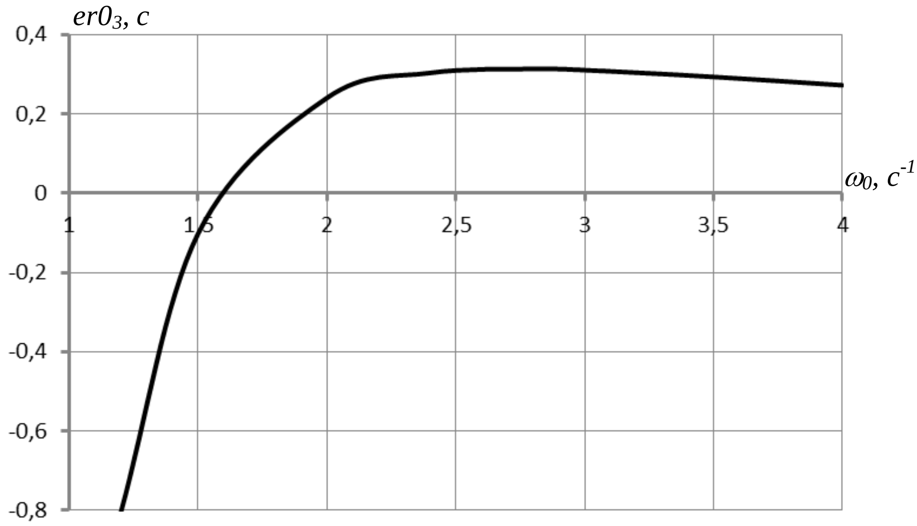

Fig. 2. The third coordinate of the error vector

The torque in the first approximation follows from the differential equation:

$$I_{RA} \cdot \ddot{\delta} + c_1 \cdot \dot{\delta} + c_0 \cdot \delta = M \ ,$$

where I_{RA} is the moment of inertia of the equivalent steering body, reduced to the axis of its rotation; , c_1, c_0 are damping factor and rigidity of the rotary device; the angle of rotation δ and its derivatives can be determined by solving equation (1), for example, by the operator method.

Despite the presence of an analytical expression $\delta(t)$ and two derivatives, the dependence of criterion (8) on the Batterwot parameter ω_0 is determined numerically (Fig. 3).

Results.

1. For the SS of rocket motion as a solid in the yaw plane without taking into account the inertia of the RA, a relation of the Batterwot parameter ω_0 to the coefficients of RL (4) was obtained.

2. Equations (7), which quantitatively characterize the accuracy of compensation of the constant perturbation acceleration depending on ω_0 were established.

3. To evaluate the quality of the transition process, an algorithm for determining the reduced performance of the RA was developed, which used an analytical solution of the differential equations (1) of the perturbed motion of the rocket.

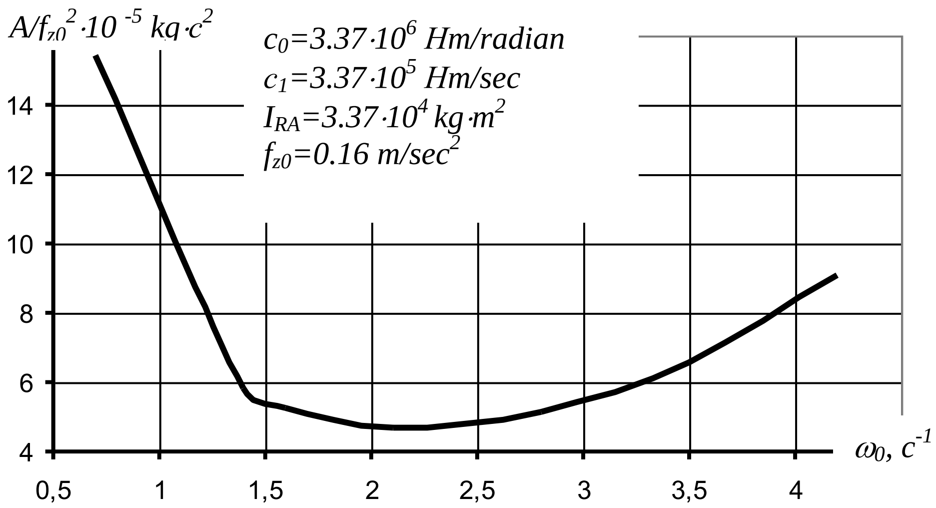

Fig. 3. The work of the RA on a transient of 6 sec duration with compensation of constant perturbation acceleration

Conclusions.

1. The coefficients of RL are proportional to the Batterwot parameter ω_0 (Fig. 1), in order to find their restriction from above it is necessary to take into account the inertia of the RA.

2. The value of ω_0 affects only the static velocity error $V_z\,(er0_3)$; in the vicinity of certain values ω_0 of $er0_3$, the value takes the smallest values closed to zero (Fig. 2).

3. The algorithm for determining the reduced operation of the RA makes it possible to choose the Batterwot parameter range ω_0 where this indicator of the quality of the transition process is minimal (Fig. 3).

The use of the described variant of the modal control method, in which the given CP roots are evenly spaced in a semicircle of radius ω_0 is an alternative possibility of calculating the RL in the development of the SS.

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