UDC 532.526 : 532.24

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SIMULATION OF THE AIR-DROPLET FLOW IN THE AIRCRAFT ICING PROBLEMS

Meteorological conditions presupposing the occurrence of icing is possible are analyzed. To describe the external air-droplet flow and the process of moisture precipitation on the streamlined aerodynamic surfaces in icing problems a model of interpenetrating media is proposed to use. This model allows one to take into account the law of droplet size distribution in an icing cloud, to consider the problem in a three-dimensional statement, and also to take into account the exchange of mechanical and thermal energy between supercooled water droplets and air flow.

Key words: *air-droplet flow, moisture precipitation on the streamlined surface, icing of aerodynamic surfaces.*

Проаналізовано метеорологічні умови, під час польоту в яких можливе виникнення обмерзання. Для опису зовнішнього повітряно-крапельного потоку і процесу осадження вологи на обтічну поверхню в задачах обмерзання аеродинамічних поверхонь літальних апаратів запропоновано використовувати модель взаємопроникних середовищ, що дозволяє враховувати закон розподілу розмірів крапель в хмарі обмерзання, розглядати задачу в тривимірній постановці, а також враховувати обмін механічною і тепловою енергією між переохолодженими водяними краплями і повітрям.

Ключові слова: *повітряно-крапельний потік, осадження вологи на обтічну поверхню, обмерзання аеродинамічних поверхонь.*

Проанализированы метеорологические условия, при полете в которых возможно возникновение обледенения. Для описания внешнего воздушно-капельного потока и процесса осаждения влаги на обтекаемую поверхность в задачах обледенения аэродинамических поверхностей летательных аппаратов предложено использовать модель взаимопроникающих сред, позволяющую учитывать закон распределения размеров капель в облаке обледенения, рассматривать задачу в трехмерной постановке, а также учитывать обмен механической и тепловой энергией между переохлажденными водяными каплями и воздухом.

Ключевые слова: воздушно-капельный поток, осаждение влаги на обтекаемую поверхность, обледенение аэродинамических поверхностей.

Introduction. In modern conditions, ensuring the safety of aircraft flights, including in adverse weather conditions, is an actual problem causes a constant interest. Under certain flying conditions, supercooled water droplets contained in clouds can freeze, falling on the aerodynamic surfaces of the aircraft, forming ice growths.

Depending on the amount, shape and location, such growths can have a significant negative influence on the aircraft: disturb the flow structure, increase turbulent wake, increase in drag, decrease in lift force and stall angle of the wing affecting adversely the stability of aircraft and its handling on the whole, lead to weight gain.

According to accepted safety standards [1], the main research instruments to be included in the certification plan for aircraft for flight in icing conditions are: flight tests in natural icing conditions, and using spray systems installed on the ahead-of-flight aircraft and creating an icing cloud; experiments in wind tunnels, both "dry" and modeling icing conditions; numerical methods. It should be noted that flight tests, although being the most reliable means of research, have significant limitations related to the danger, as well as the difficulty of ensuring the reproduction of experimental conditions, in addition, they require significant financial and time costs. Ground-based experimental methods also require the use of expensive and complex cooled high-speed wind tunnels equipped with a system for the reproduction of icing conditions. Such methods, although they make it possible to significantly expand the range of the investigated parameters, but, in turn, also do not give a complete idea of the distribution of the parameters of the air-droplet flow in the investigated region. In addition, experiments conducted under ground conditions can not accurately reproduce the icing conditions in flight, requiring the use of scaled models. As a result, in order to reduce the financial and time cost of developing anti-icing systems, to assess their effectiveness, to understand the effects of changing the geometry of the aerodynamic surfaces due to the formation of ice build-up on the flow field and, accordingly, to create the most advanced anti-icing systems in modern conditions, there is a need to apply numerical simulation methods.

Until now, a number of well-known techniques and software products have been developed in various countries to simulate the icing processes (LEWICE, ONERA, CANICE, etc.) [2-4], in which, as a rule, the external airflow is described using the potential equations, and motion of supercooled water droplets – using a trajectory model. Also, the application of the trajectory model in combination with the Navier-Stokes equations of compressible gas for the describing of the air-droplet flow is considered in [5-6]. When implementing this approach, the airflow parameters are first calculated, along which a number of trajectories of supercooled droplets with a small margin of "enveloping" a streamlined profile are built. The concentration of droplets directly at the surface and, correspondingly, the local collection efficiency for the streamlined profile, is determined by the condition for the conservation of the mass flow of droplets in the cross section, limited by neighboring droplet trajectories.

It should be noted that the traditional approach using the trajectory model [2-4] does not take into account the mutual exchange of mechanical and thermal energy between supercooled droplets and air flow, is associated with certain difficulties in implementation of three-dimensional statement of the problem: in the case of complex geometry of streamlined bodies, multi-body configurations.

Objective of the paper is to develop a mathematical model describing the motion of the air-droplet flow taking into account the mutual exchange of mechanical and thermal energy between supercooled water droplets and airflow in a three-dimensional statement.

Physical Problem Statement. All observed weather phenomena, including meteorological conditions, in which the occurrence of icing is possible, are formed in the troposphere at heights of up to 7 km at the poles and up to 16 km at the equator. The powerful thermal vertical streams of moist air, the continuous mixing of air masses with different temperatures and pressures, the lowering of the temperature and air pressure with altitude, also cause a change in the water concentration in the air, condense water vapor and form clouds, fog, rain, snow or hail. It is known that under certain conditions, water droplets, contained in clouds or rain at a negative temperature of the ambient air, can be in a supercooled state [7]. The icing of aircraft in most cases occurs during a flight in this environment.

The main meteorological parameters on which the intensity of icing depends are:

- the amount of condensed water per unit cloud volume (liquid water content, LWC);
- the air temperature (T);
- the size of the water droplets.

Water content varies considerably with temperature and can fluctuate strongly for both the same cloud type and the same cloud. In practice, usually used water content averaged over large areas, equivalent in volume of $1\ m^3$ and a length of several kilometers. As the temperature decreases, the water content decreases, and the probability of icing decreases. It should also be noted that an icing cloud represents a polydisperse aerosol, which contains supercooled water droplets of different sizes – from a few up to tens of microns, and during rain up to hundreds of microns. To estimate the size of droplets contained in the cloud, in practice, *the mean volume diameter* (*MVD*) is often used, determined in such a way that 50% of the total water droplets contained in a unit of cloud volume have a diameter larger, and 50% – smaller than *MVD*.

It is believed that the major types of clouds, which may cause icing are stratiform and cumuliform clouds and also freezing drizzle or freezing rain conditions (in the presence in atmosphere of so-called *supercooled large droplets*, *SLD*). Herewith the range of meteorological parameters presupposing the occurrence of icing is determined by the Aviation Rules of the International Aviation Committee part 25 [8] and also Title 14 of the Code of Federal Regulations, part 25, Appendices C and O [1], which also regulate the law of distribution of sizes of supercooled water droplets in the cloud.

Mathematical Problem Statement. When describing the external air-droplet flow in a three-dimensional statement, a model of interpenetrating media proposed by H.A. Rakhmatullin [9-11] was used. This model is based on the concept of a multiple-velocity/multiple-temperature medium, with each point of the medium being characterized by as many velocities and temperatures as the amount of layers that the considered medium contains. Each phase continuously fills the space.

The main assumptions of the model are the following:

- the medium is assumed to be multiple-velocity and multiple-temperature, consisting of air – viscous compressible carrying gas and supercooled water droplets – fractions of incompressible spherical particles of specified diameter with no interaction between them;
- heat capacities of the air and droplets are constant;
- the droplets sizes are many times larger than the molecular-kinetic distances and many times smaller than the distances, and the averaged macroscopic parameters of flow vary significantly;
- the interaction of the supercooled water droplets and the carrying air is taken into account via air viscosity;
- the temperature of a single droplet is constant throughout its volume.

The equations that describe the carrying air flow and supercooled water droplets are coupled through the source terms, taking into account the momentum and energy exchange between states. They differ from the Navier-Stokes equations only by the presence of these sources. In order to specify the terms that describe the interphase interaction, the results of studies of the processes occurring at the gas flow over particles are used [12].

The unsteady equations of the multi-phase flow are written as follows [9-11]:

$$\frac{\partial \hat{q}}{\partial t}+\frac{\partial \hat{E}}{\partial \xi}+\frac{\partial \hat{F}}{\partial \eta}+\frac{\partial \hat{G}}{\partial \zeta}+\hat{H}=\frac{1}{\mathrm{Re}}\left(\frac{\partial \hat{R}}{\partial \xi}+\frac{\partial \hat{S}}{\partial \eta}+\frac{\partial \hat{T}}{\partial \zeta}\right), \tag{1}$$

where

$$\hat{q}=\frac{1}{J}\begin{bmatrix}\rho\\ \rho\mathrm{u}\\ \rho\mathrm{v}\\ \rho\mathrm{w}\\ \mathrm{e}\\ \rho_{\mathrm{jk}}\\ \rho_{\mathrm{jk}}\mathrm{u}_{\mathrm{jk}}\\ \rho_{\mathrm{jk}}\mathrm{v}_{\mathrm{jk}}\\ \rho_{\mathrm{jk}}\mathrm{w}_{\mathrm{jk}}\\ \mathrm{e}_{\mathrm{jk}}\end{bmatrix},\quad \hat{E}=\frac{1}{J}\begin{bmatrix}\rho U\\ \rho Uu+\xi_x p\\ \rho Uv+\xi_y p\\ \rho Uw+\xi_z p\\ (e+p)U\\ \rho_{jk}U_{jk}\\ \rho_{jk}U_{jk}u_{jk}\\ \rho_{jk}U_{jk}v_{jk}\\ \rho_{jk}U_{jk}w_{jk}\\ e_{jk}U_{jk}\end{bmatrix},\quad \hat{F}=\frac{1}{J}\begin{bmatrix}\rho V\\ \rho Vu+\eta_x p\\ \rho Vv+\eta_y p\\ \rho Vw+\eta_z p\\ (e+p)V\\ \rho_{jk}V_{jk}\\ \rho_{jk}V_{jk}u_{jk}\\ \rho_{jk}V_{jk}v_{jk}\\ \rho_{jk}V_{jk}w_{jk}\\ e_{jk}V_{jk}\end{bmatrix},\quad \hat{G}=\frac{1}{J}\begin{bmatrix}\rho W\\ \rho Wu+\zeta_x p\\ \rho Wv+\zeta_y p\\ \rho Ww+\zeta_z p\\ (e+p)W\\ \rho_{jk}W_{jk}\\ \rho_{jk}W_{jk}u_{jk}\\ \rho_{jk}W_{jk}v_{jk}\\ \rho_{jk}W_{jk}w_{jk}\\ e_{jk}W_{jk}\end{bmatrix}\quad \hat{H}=\frac{1}{J}\begin{bmatrix}0\\ H_u\\ H_v\\ H_w\\ H_e\\ 0\\ -H_{uk}\\ -H_{vk}\\ -H_{wk}\\ -H_{ek}\end{bmatrix},$$

$$\hat{R}=\frac{1}{J}\begin{bmatrix}0\\ \mu\left(\xi_x^2+\xi_y^2+\xi_z^2\right)u_\xi+\left(\frac{\mu}{3}\right)\xi_x\left(\xi_x u_\xi+\xi_y v_\xi+\xi_z w_\xi\right)\\ \mu\left(\xi_x^2+\xi_y^2+\xi_z^2\right)v_\xi+\left(\frac{\mu}{3}\right)\xi_y\left(\xi_x u_\xi+\xi_y v_\xi+\xi_z w_\xi\right)\\ \mu\left(\xi_x^2+\xi_y^2+\xi_z^2\right)w_\xi+\left(\frac{\mu}{3}\right)\xi_z\left(\xi_x u_\xi+\xi_y v_\xi+\xi_z w_\xi\right)\\ \frac{k}{\Pr(\gamma-1)}\left(\xi_x^2+\xi_y^2+\xi_z^2\right)\frac{\partial}{\partial\xi}\left(a^2\right)+\\ +\frac{\mu}{2}\left(\xi_x^2+\xi_y^2+\xi_z^2\right)\left(u^2+v^2+w^2\right)_\xi+\\ +\frac{\mu}{6}\left[\xi_x^2\left(u^2\right)_\xi+\xi_y^2\left(v^2\right)_\xi+\xi_z^2\left(w^2\right)_\xi+2\xi_x\xi_y(uv)_\xi\right]\\ 0\\ 0\\ 0\\ 0\end{bmatrix},\ \hat{S}=\frac{1}{J}\begin{bmatrix}0\\ \mu\left(\eta_x^2+\eta_y^2+\eta_z^2\right)u_\eta+\left(\frac{\mu}{3}\right)\eta_x\left(\eta_x u_\eta+\eta_y v_\eta+\eta_z w_\eta\right)\\ \mu\left(\eta_x^2+\eta_y^2+\eta_z^2\right)v_\eta+\left(\frac{\mu}{3}\right)\eta_y\left(\eta_x u_\eta+\eta_y v_\eta+\eta_z w_\eta\right)\\ \mu\left(\eta_x^2+\eta_y^2+\eta_z^2\right)w_\eta+\left(\frac{\mu}{3}\right)\eta_z\left(\eta_x u_\eta+\eta_y v_\eta+\eta_z w_\eta\right)\\ \frac{k}{\Pr(\gamma-1)}\left(\eta_x^2+\eta_y^2+\eta_z^2\right)\frac{\partial}{\partial\eta}\left(a^2\right)+\\ +\frac{\mu}{2}\left(\eta_x^2+\eta_y^2+\eta_z^2\right)\left(u^2+v^2+w^2\right)_\eta+\\ +\frac{\mu}{6}\left[\eta_x^2\left(u^2\right)_\eta+\eta_y^2\left(v^2\right)_\eta+\eta_z^2\left(w^2\right)_\eta+2\eta_x\eta_y(uv)_\eta\right]\\ 0\\ 0\\ 0\\ 0\end{bmatrix},\qquad (2)$$

$$\hat{T}=\frac{1}{J}\begin{bmatrix}0\\ \mu\left(\zeta_x^2+\zeta_y^2+\zeta_z^2\right)u_\zeta+\left(\frac{\mu}{3}\right)\zeta_x\left(\zeta_x u_\zeta+\zeta_y v_\zeta+\zeta_z w_\zeta\right)\\ \mu\left(\zeta_x^2+\zeta_y^2+\zeta_z^2\right)v_\zeta+\left(\frac{\mu}{3}\right)\zeta_y\left(\zeta_x u_\zeta+\zeta_y v_\zeta+\zeta_z w_\zeta\right)\\ \mu\left(\zeta_x^2+\zeta_y^2+\zeta_z^2\right)w_\zeta+\left(\frac{\mu}{3}\right)\zeta_z\left(\zeta_x u_\zeta+\zeta_y v_\zeta+\zeta_z w_\zeta\right)\\ \frac{k}{\Pr(\gamma-1)}\left(\zeta_x^2+\zeta_y^2+\zeta_z^2\right)\frac{\partial}{\partial\zeta}\left(a^2\right)+\\ +\frac{\mu}{2}\left(\zeta_x^2+\zeta_y^2+\zeta_z^2\right)\left(u^2+v^2+w^2\right)_\zeta+\\ +\frac{\mu}{6}\left[\zeta_x^2\left(u^2\right)_\zeta+\zeta_y^2\left(v^2\right)_\zeta+\zeta_z^2\left(w^2\right)_\xi+2\zeta_x\zeta_y(uv)_\zeta\right]\\ 0\\ 0\\ 0\\ 0\end{bmatrix},$$

$$\mathrm{e}=\rho\left[\varepsilon+\frac{1}{2}\left(\mathrm{u}^2+\mathrm{v}^2+\mathrm{w}^2\right)\right],\ e_{jk}=\gamma\rho_{jk}\left[\omega T_{jk}+\left(u_{jk}^2+v_{jk}^2+w_{jk}^2\right)\right].\qquad (3)$$

The following notations are used in the equations: u, v, w – are the velocity components in the directions x, y, z; ρ, p, e – are the density, pressure, and total energy of the unit volume of air; a – is the sound velocity; γ – is the specific heat ratio; μ – is the dynamic viscosity coefficient; Re – is the Reynolds number; $\Pr$ – is the Prandtl number; $\xi_x, \xi_y, \xi_z, \eta_x, \eta_y, \eta_z, \zeta_x, \zeta_y, \zeta_z, J$ – are the metric coefficients and Jacobian of the coordinate transforms $\xi = \xi(x, y, z, t), \eta = \eta(x, y, z, t), \zeta = \zeta(x, y, z, t)$, respectively. U, V, W – are the contravariant components of the velocity vectors of the carrying air and supercooled droplets; variables with subscript *j* describe the particles; ω – is the specific heat ratio of the droplets and air at a constant pressure, subscript *k* varies from *1* to *n*, where *n* is the number of intervals that correspond to the mass fractions of droplets of a given size in the air-droplet flow (according to the accepted law of droplet size distribution by volume of the cloud).

The magnitude of the momentum and energy exchange between the layers is specified as [12]:

$$H_{uk} = \rho_{jk} A_{jk} \left(u - u_{jk}\right),\; H_{vk} = \rho_{jk} A_{jk} \left(v - v_{jk}\right), H_{wk} = \rho_{jk} A_{jk} \left(w - w_{jk}\right), H_{ek} = \rho_{jk} A_{jk} B_{jk}, \tag{4}$$

$$H_u = \sum_{k=1}^{n} H_{uk},\; H_v = \sum_{k=1}^{n} H_{vk},\; H_w = \sum_{k=1}^{n} H_{wk},\; H_e = \sum_{k=1}^{n} H_{ek},$$

where A_{jk}, B_{jk} are dimensionless coefficients determining the momentum and energy exchange,

$$A_{jk} = \frac{9}{2} \frac{\mu_g f_{jk}}{m_{jk} r_{jk}^2} \frac{\bar{L}}{\bar{V}},$$

$$B_{jk} = 2\gamma \left[\bar{q}_{jk} \Delta \bar{q}_{jk} - g_c \left(T_{jk} - T\right)\right], \tag{5}$$

$$\bar{q}_{jk} \Delta \bar{q}_{jk} = u_{jk} \left(u - u_{jk}\right) + v_{jk} \left(v - v_{jk}\right) + w_{jk} \left(w - w_{jk}\right), g_c = Nu_{jk} / 6 f_{jk} \Pr. \tag{6}$$

Normalized friction coefficient f_j is calculated by the following formula:

$$f_{jk} = \frac{C_D}{C_{\mathrm{Dstokes}_k}}, \tag{7}$$

where C_D – is the drag coefficient of the drops, and

$$C_{D\mathrm{stokes}_k} = \frac{24}{\mathrm{Re}_{jk}}.$$

Re_{jk}, is determined in terms of the relative drop velocity by:

$$\left|\Delta q_{jk}\right| = \sqrt{\left(u-u_{jk}\right)^2+\left(v-v_{jk}\right)^2+\left(w-w_{jk}\right)^2}\,, \tag{8}$$

was calculated using the formula:

$$\mathrm{Re}_{jk} = \frac{2\left|\Delta q_{jk}\right| r_{jk}\rho}{\mu_g}\,. \tag{9}$$

In the calculations, the normalized friction coefficient and Nusselt number for the drops are determined as follows:

$$f_{jk} = \begin{cases} 1, \mathrm{Re}_{jk} \le 0{,}49, \\ 1{,}125 \quad \mathrm{Re}_{jk}^{0{,}163}, 0{,}49 \le \mathrm{Re}_{jk} \le 80, \\ 0{,}0125 \quad \mathrm{Re}_{jk}^{1{,}217}, 80 \le \mathrm{Re}_{jk} \le 1000, \\ \mathrm{Re}_{jk}/12, \mathrm{Re}_{jk} > 1000, \end{cases} \tag{10}$$

$$\mathrm{Nu} = 2 + 0{,}459\,\mathrm{Re}^{0{,}55}\,\mathrm{Pr}^{0{,}33}\,. \tag{11}$$

In the relations (4) – (11) $\bar{L}$ – is a typical length scale; r_{jk} – is the drop radius, corresponding to the *k-th* interval; μ_g – is the dynamic viscosity coefficient of the carrying air; m_{jk} – mass density of droplets, corresponding to the *k-th* interval; $\bar{V}$ – is the datum speed; ρ – is the dimensionless density of the carrying air; and the remaining notations are universally accepted.

The initial distribution of the droplet concentration in the air-droplet flow, corresponding to the *k-th* interval:

$$\rho_{jk} = \rho\phi_k/\left(1-\phi_k\right), \tag{12}$$

where $\phi_k = W_{jk}/W_m$ – the mass fraction of droplets in the air-droplet flow having dimensions, corresponding to the *k-th* interval.

Conclusions: A mathematical model that can be used to solve the problem of icing of aircraft aerodynamic surfaces during the flight in adverse meteorological conditions has been developed. The model makes it possible to describe the motion of the air-droplet flow taking into account the law of the droplet size distribution in the icing cloud, the mutual exchange of mechanical and thermal energy between the supercooled water droplets and the air flow, and proceed to the solution of the problem in a three-dimensional statement.

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