

Spatially distributed GIS-realized mathematical model of rainstorm erosion losses of soil

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Received: 07.02.2019 Received in revised form: 25.02.2019 Accepted: 29.03.2019 **Abstract.** In connection with the wide and ever increasing spread of erosion degradation of agricultural lands in Ukraine, the task of developing mathematical models and methods for calculating water erosion of soils corresponding to the current level of erosion study and the demands of soil protection practices is becoming increasingly important. The article is

devoted to the development of a spatially distributed GIS-implemented mathematical model of rainstorm soil erosion, which accounts for most of the annual soil losses (in the Steppe zone, for example, about 90 %). The development of the model is based on the most theoretically and informationally grounded model for the Steppe and Forest-Steppe of Ukraine, "the logical-mathematical model of rainstorm soil outwash" developed by H. I. Shvebs (1974, 1981), as well as the results of theoretical and field studies and mathematical modeling of the slope runoff and water erosion of soil, carried out at the Department of Physical Geography and Environmental Management of Odessa I. I. Mechnikov National University in the 1990s - 2010s, and also the possibilities of modern geoinformation technologies. For the spatial implementation of the model, a raster model of spatial data and operators of the PCRaster GIS-package (University of Utrecht, the Netherlands) were used, integrated with the Basic programming language into a single system that provides an implementation of the computational algorithm. The developed physical-statistical model of soil erosion-sedimentation takes into account the peculiarities of the formation of slope runoff and soil outwash in conditions of excessive nonstationarity of heavy rainfall, as well as spatial heterogeneity of all major natural and economic factors of water erosion on a slope, including slope steepness, exposure, longitudinal and transverse forms of slopes, soil erodibility, structure of sown areas and anti-erosion measures. Checking the adequacy of the mathematical model was performed using observational data of four experimental catchments; two runoff plots of the Moldavan water-balance station with total area of 0.08 ha, the Ploska catchment with area of 8.5 ha (Boguslav field experimental base of Ukrainian Hydrometeorological Institute) and the Sukha catchment with area of 63 ha (Veliko-Anadol water-balance station) with observation periods of 17-31 years. Comparison of the calculated average over the catchment area of mean annual values of rainstorm soil losses, with the corresponding values obtained from measurements on these catchments, made on the basis of Nash-Sutcliff efficiency criterion (NS), allowed us to evaluate the quality of the model as good (NS = 0.72).

Keywords: water soil erosion, mathematical model, GIS-implementation, model verification

Просторово-розподілена ГІС-реалізована математична модель зливових ерозійних втрат ґрунту

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Анотація. У зв'язку із широким і постійно зростаючим розповсюдженням ерозійної деградації сільськогосподарських земель в Україні, все більш актуальною є задача розробки математичних моделей і методів розрахунків водної ерозії грунтів, відповідно до сучасного рівня розвитку ерозієзнавства та запитам практики грунтозахисного землеробства. Стаття присвячена розробці просторово-розподіленої ГІС-реалізованої математичної моделі зливових ерозійних втрат ґрунту, на частку яких припадає більша частина (у степовій зоні – близько 90 %) річних втрат грунту. в основу розробки моделі покладена «логіко-математична модель зливового змиву ґрунту», розроблена Г. І. Швебсом (1974, 1981), а також результати теоретичних і польових досліджень та математичного моделювання схилового стоку і водної ерозії ґрунтів, виконаних на кафедрі фізичної географії і природокористування Одеського національного університету імені І. І. Мечникова у 1990-і – 2010-і роки, а також можливості сучасних геоінформаційних технологій. Для просторової реалізації моделі використана растрова модель просторових даних і оператори ГІС-пакету РСRaster (Університет м. Утрехта, Нідерланди), інтегровані за допомогою мови програмування Basic у єдину систему, що забезпечує реалізацію розрахункового алгоритму. Розроблена фізико-статистична модель змиву-акумуляції грунтув враховує особливості формування силового стоку та змиву ґрунту в умовах підвищеної не стаціонарності випадіння зливових опадів, а також просторову неоднорідність всіх основних природних та господарських факторів водної ерозії на схилі – ухилу схилу, експозиції, поздовжньої та поперечної форми схилів, протиерозійної стійкості ґрунтів, структури посівних площ і протиерозійних заходів. Перевірка адекватності математичної моделі виконана з використанням даних спостережень у межах чотирьох експериментальних водозборів із площами 0,08 га (дві стокові ділянки Молдавської водно-балансової станції), водозбір улоговина Плоска із площею 8,5 га (Богуславська польова експериментальна база Гідрологічного інституту України) і водозбір Сухий із площею 63 га (Велико-Анадольська водно-балансова станція) із періодами спостережень 17-31 рік. Зіставлення розрахованих осереднених за площею середньо багаторічних значень зливових ерозійних втрат ґрунту із відповідними значеннями, отриманими за даними вимірювань на цих водозборах, виконане на основі критерію ефективності Неша-Саткліффа (NS), дозволило оцінити якість моделі як добру (NS=0,72).

Ключові слова: водна ерозія трунту, математична модель, ГІС-реалізація, верифікація моделі

Introduction. In accordance with (Natsionalna dopovid...., 2010), the area of eroded land in Ukraine is about 16 million hectares, which is 38.4% of the area of agricultural land .The area of eroded arable land is about 13 million hectares or 39.9% of its square. In many administrative regions, especially in the Steppe zone, these figures are significantly higher. So, in the Dnipropetrovsk region 43.8% of the agricultural land area is eroded, in Odessa region -46.8%, in Kirovograd region -54.0%, in Lugansk region – 71.8%, in Donetsk region – 85.9%. Over the past decades, the area of eroded land in the country has been constantly increasing and this is largely due to the growth of the areas of mostly medium and strongly eroded soils. As noted by the authors (Kanash & Osipchuk, 2003), a steady trend has arisen towards increase in the area of medium eroded soils by an average of 18.9 thousand hectares, strongly eroded soils - by 5.2 thousand hectares a year. The erosion degradation of soils is associated with the deterioration of their water-physical and chemical properties, biological activity and decrease in fertility. Products of erosion destruction of the soil cover are the process of siltation of river valleys, deterioration of the quality of surface water, etc. The final result of the development of soil erosion is desertification.

With the spread of soil water erosion in almost all natural zones of Ukraine, the permanent expansion of eroded land, the problem of protecting agricultural landscapes from the damaging effects of soil erosion is becoming increasingly important. The designing of anti-erosion measures is based on mathematical models of erosion losses of soil. The practice of conservation agriculture in the United States and many other countries is based on the use of the modern version of the Universal Soil Loss Equation (RUSLE) (Renard et al., 1991) and its modifications - MUSLE, USLE-M, RUSLE-3D. Other mathematical models are also used, in particular, KINEROS and KINEROS2 (Smith et al., 1995), LISEM (De Roo et al., 1996), WEPP (Ascough et al., 1997), EUROSEM (Morgan et al., 1998), EROSION 2D/3D (Schindewolf & Schmidt, 2012). However, for the natural-economic conditions

of Ukraine, these models are not parameterized, often do not have the necessary information support, and have not been verified.

In the 1970s - 90s in Ukraine a number of mathematical models of soil erosion losses were developed with varying degrees of theoretical validity and information provision, which were used or recommended for anti-erosion design purposes. Among them, the most famous are: "logicalmathematical model of erosion losses of soil", developed by H. I. Shvebs (Shvebs, 1974, 1981), "mathematical-statistical model of soil erosion losses during heavy rainfall", developed in the former Ukraine Research Institute for Soil Erosion Protection (Lavrovskiy et al, 1987; Spravochnik..., 1990) and the formula of I. K. Sribnyi (Sribny, 1977; Sribny & Vergunov, 1993). It should be noted that all these models belong to the category of models with lumped parameters, that is, they operate with average values of input data for the territory under consideration (length and slope, characteristics of the soil cover, agricultural background and antierosion measures), and the result of the calculation is the average value of soil erosion rate for the area. At present, it is generally recognized that the optimal system of anti-erosion measures is a soil-protective landscape-adaptive farming system, taking into account the specific features of the structure and functioning of the agrolandscape. The design of such systems requires the use of spatially distributed mathematical models of soil erosion, taking into account the spatial differentiation of all the main factors of the slope erosion-sedimentation process.

The development of such a model was performed during the 1990s - 2010s at the Odessa I. I. Mechnikov National University, Department of Physical Geography and Environmental Management. The development is based on the logical-mathematical model by H. I. Shvebs (Shvebs, 1974, 1981), as well as on the basis of results of researches and modeling of storm soil erosion and surface runoff formation under conditions of their pronounced nonstationarity on slopes of complex longitudinal and transverse shape using the capabilities of modern geo-information technologies (Svetlitchnyi, 1995, 1999; Svetlitchnyi et al., 2003; Pyatkova, 2008, 2013). The resulting spatial version of the model is a physical-statistical mathematical model of erosion-sedimentation, which makes it possible to calculate for a given point of the slope not only the average long-term amount of erosion losses of soil, but also deposition of sediments.

The purpose of this article is to summarize the results of research on the spatial GIS implementation of the model on slopes of complex transverse shape and model verification using long-term monitoring of sediment runoff on catchments of various sizes. *The object* of research is the slope water-erosion process, which causes erosion degradation of agro-landscapes. *The subject* of research is spatial implementation and verification of the physical-statistical mathematical model of soil erosion losses using geo-information (GIS) technologies.

Material and Methods. The article is based on the results of theoretical and field studies and mathematical modeling of water soil erosion, performed at Odessa I. I. Mechnikov National University (Shvebs, 1974, 1981; Svetlitchnyi, 1995, 1999, Chorny, 1996; Svetlitchnyi et al., 2004; Pyatkova, 2008, 2013)

For the spatial GIS-implementation of the model, a raster spatial data model, analytical capabilities of the PCRaster Environmental Modeling Package (GISpackage) (PCRaster..., 2018) and the programming language Basic were used.

The verification of the model was carried out using the materials of long-term observations on the slope watersheds and runoff plots at runoff and waterbalance stations of the Hydrometeorological Service of Ukraine and Republic of Moldova.

The methods used are comparative-geographical, statistical, graph-analytical, geographic information mapping and mathematical modeling,

Results and Discussion. GIS-implementation of the model on the basis of raster model of spatial data assumes that each cell of the digital raster map with *i*, *j* coordinates is considered as the outlet of the catchment of this cell. The catchments of individual cells, which in the environment of the PCRaster package identified as "cell upstream elements", we shall call "partial catchments" or "sub-catchments".

Calculated expressions of the spatial version of the physical-statistical mathematical model of soil erosion-sedimentation in this case take the form:

$$\text{for } x \leq L_{-}$$

$$W_{S}(i, j) = 2,6 \cdot 10^{-6} \Biggl[\Biggl(1 + 0,5 \Biggl(\frac{x'}{x} \Biggr)^{0,5} \Biggr) K_{HM}(i, j) j_{R}(i, j) I^{m}(i, j) f_{a}(i, j) x^{0,5} + K_{HM}(i, j) j_{R}(i, j) I^{m}(i, j) f_{a}(i, j) x^{1,5} + K_{HM}(i, j) j_{R}(i, j) \frac{dI^{m}(i, j)}{dn} f_{a}(i, j) x^{1,5} + K_{HM}(i, j) j_{R}(i, j) \frac{dI_{R}(i, j)}{dn} f_{a}(i, j) x^{1,5} + K_{HM}(i, j) \frac{dj_{R}(i, j)}{dn} I^{m}(i, j) f_{a}(i, j) x^{1,5} + \frac{dK_{HM}(i, j)}{dn} j_{R}(i, j) I^{m}(i, j) f_{a}(i, j) x^{1,5} + K_{HM}(i, j) j_{R}(i, j) I^{m}(i, j) f_{a}(i, j) x^{1,5} + K_{HM}(i, j) j_{R}(i, j) I^{m}(i, j) f_{a}(i, j) x^{1,5} + K_{HM}(i, j) j_{R}(i, j) I^{m}(i, j) f_{a}(i, j) x^{1,5} - \frac{d(x'^{0,5})}{dn} \Biggr],$$

for $x > L_a$

$$\begin{split} W_{S}(i,j) &= 2,6 \cdot 10^{-6} \left[\left(1 + 0,5 \left(\frac{x'}{x} \right)^{0,5} \right) K_{HM}(i,j) j_{R}(i,j) I^{m}(i,j) f_{a}(i,j) L_{\Delta}^{0,5} + K_{HM}(i,j) j_{R}(i,j) J_{R}(i,j) I^{m}(i,j) I^{m}(i,j) \frac{df_{a}(i,j)}{dn} x + K_{HM}(i,j) j_{R}(i,j) \frac{dI^{m}(i,j)}{dn} f_{a}(i,j) x + K_{HM}(i,j) \frac{dj_{R}(i,j)}{dn} I^{m}(i,j) f_{a}(i,j) x + \frac{dK_{HM}(i,j)}{dn} j_{R}(i,j) I^{m}(i,j) f_{a}(i,j) x + K_{HM}(i,j) j_{R}(i,j) I^{m}(i,j) f_{a}(i,j) x + K_{HM}(i,j) J_{R}(i,j) I^{m}(i,j) f_{a}(i,j) x + K_{HM}(i,j) J_{R}(i,j) I^{m}(i,j) J_{a}(i,j) x + K_{HM}(i,j) J_{R}(i,j) I^{m}(i,j) f_{a}(i,j) x + K_{HM}(i,j) J_{R}(i,j) I^{m}(i,j) f_{a}(i,j) x + \frac{d(x'^{0,5})}{dn} \right], \end{split}$$

where $W_s(i, j)$ is a long-term average soil losses, t/ha/yr, in the cell with coordinates (i, j); x is the distance from the watershed to the cell (i, j) along the local drain lines, m; x' is the modified distance from the watershed to the cell (i, j) along the local drain lines, m; $K_{HM}(i, j)$ is an average value of the hydrometeorological factor of rainstorm erosion of soil within the sub-catchment of the cell (i, j); $j_{R}(i, j)$ is an average value of the characteristics of relative soil erodibility within the sub-catchment of the cell (i, j); $f_a(i, j)$ is an average value of the agro technical factor within subcatchment of the cell (i, j); I(i, j) is an average slope steepness within the slope micro catchment above the cell (i, j), ∞ ; L_a is the length of the zone of increase in the intensity of soil washout which is adjacent to watershed of the slope, m; L_{Λ} is the length of the zone of increase in the intensity of soil washout within the sub-catchment of the cell (i, j) when $x > L_a$, m; m is the exponent at a slope steepness, for non-eroded and light eroded chernozems and forest soils equal to 1.30, for medium and strongly eroded equal to 1.35.

The modified distance from the watershed to the cell (i, j) along the local drain line x' is calculated by the equation:

$$x' = 0.5x \left[K_f(i, j) + 1 \right], \tag{3}$$

where K_f is the coefficient of development of the form of sub-catchments which takes into account the degree of concentration of surface flows, dimensionless.

The coefficient of development of a subcatchment's form characterizes the degree of deviation of the width of the sub-catchment from some average value at a given length of the sub-catchment ("standard") for the given geomorphological and soil-climatic conditions. It is determined by the ratio of the sub-catchment's width B of a given cell with coordinates (i, j) to the width of a standard subcatchment the same length B_{er} .

The inclusion of the coefficient of development of the sub-catchment's form into expressions (1) - (2) allows us to take into account the structure of surface flows. First, the value of the coefficient K_f characterizes a greater ($K_f > 1$) or lesser ($K_f < 1$) concentration of surface flows with respect to the "standard" conditions, increasing or decreasing the modified slope length x'. To an even greater degree, the concentration of surface flows is taken into account by calculating the first derivative of the modified length of the slope along the streamline. If the value of the derivative is positive, the transverse concentration of surface flows increases, if the value is negative, the concentration of flows decreases. This is especially important for the so-called converging slopes. However, experience has shown that in order to take into account the concentration of surface flows, in this case, a detailed hydrologically correct DEM is necessary and that the raster cell size does not exceed 10 m.

The value of the L_a , as the length of the zone of growth of intensity of sediments formation from the watershed down the slope is calculated by the formula arising from the formula of the rate of surface flows from the normative document (Pravila..., 1987)

$$L_a = 0.854 k_v m_c^{\frac{3}{2}} \left(r_{\tau, p\%} \varphi \, b_c \, I_a \right)^{\frac{1}{2}}, \qquad (4)$$

where k_v is the coefficient taking into account the discrepancy between the velocity of flow and velocity of runoff wave, equal to 1.5; $r_{10,p\%}$ is the maximum average intensity of the 10% probability of rainfall for a ten-minute time interval, mm/min, for Steppe and Forest Steppe of Ukraine equal to 2.1 mm/min; m_c is a surface roughness coefficient, dimensionless; φ is the runoff coefficient, dimensionless; b_c is an average width of sub-catchment, m; I_a is an average slope steepness, ‰.

The flowchart of the GIS-realized model is presented in Fig. 1. The model consists of several blocks within which the corresponding calculations are performed.

Input data for calculations of average annual rainstorm erosion soil losses within the study area is:

1) hydrologically correct digital elevation model;

2) map of slope's watersheds;

3) map of genetic types and subtypes of soils;

4) eroded soils distribution map;

5) maps of land use with borders of crop rotation fields, information on crops and technologies of their cultivation and erosion control measures;

6) value of the norm of the hydrometeorological factor of rainstorm soil losses, obtained on the basis of observation data at the nearest meteorological station or taken from the map (Svetlitcnhyi et al.,2004);

7) value of moisture of the half-meter soil layer at the flat place according to observations of nearest agrometeorological station.

The soil moisture map is calculated using a sub model presented in (Svetlitchny et al., 2003). The model is founded on the dependence of the soil moisture on the form of the slope, its exposition and remoteness from the divide. The model is fully implemented in the environment of the PCRaster package.

The transition from soil moisture to the index of

antecedent moisturizing which is required to calculate the hydrometeorological factor of soil losses (K_{HM}), is carried out with the help of dependence obtained using data of observations for soil moisture at agrometeorological stations of Ukraine (Svetlitchny & Ivanova 2004). The dependence between the moisture of the upper half-meter soil layer (W) and index of antecedent moisturizing (I_W) is described by

$$I_{W} = 59,2 \left(\frac{W - W_{MH}}{W_{MC} - W_{MH}} \right) + \frac{71,3 - W_{MH}}{4,08}, \quad (5)$$

the following expression:

where W_{MC} is the field capacity and W_{MH} is the maximum hygroscopicity of the upper half-meter layer of soil, mm.

At each calculation step, the values of variables I, $j_{R'}$, $K_{HM'}$, f_a , m_c , φ , b_c are averaged within the subcatchment of the given cell. The problem of variable averaging is solved under GIS-implementation of the model by using the Basic programming language integrated with operators of the PCRaster package.

In this way, the GIS-implemented spatiallydistributed physical-statistical model of rainstorm erosion-sedimentation of soil allows us to take into account the main features of the formation of rainstorm surface runoff, the influence of steepness, length, and longitudinal and transverse curvature of slopes as well as the spatial variability of natural and anthropogenic factors of soil erosion on slopes.

Verification, that is, in this case, the establishment of its truth, adequacy, is a necessary and mandatory step in the development of any mathematical models. However, it should be noted that, as applied to mathematical models designed to estimate the norm (mean multiyear value) of soil erosion losses, this rule is not always fulfilled. The problem is that in Ukraine there is not a sufficient number of field stations with long-term and reliable observations of



Fig. 1. Flowchart of calculation of average long-term rate of rainstorm soil erosion-sedimentation

the erosion process on slopes, such as, for example, in the USA and in some other countries. Practically the only source of such data is observations at Runoff and Water-Balance Stations at runoff plots and small catchments without pronounced thalweg or with thalweg, the slope of which is not significantly different from the slope of the catchment. On such catchments, sediment yield is recorded only in their outlets and, from the point of view of evaluating models of erosion soil losses on slopes, it is important that deposition of slope sediments does not occur in thalwegs, which is inevitable if the slope of thalweg is substantially less than the slope of adjacent slopes. territory of Republic of Moldova, showed that from the experimental catchments with long periods of observation of the storm-washout of the soil, only the catchment Ploska of Boguslav FEHB, catchment Sukha of Veliko-Anadol WBS and runoff plots No.1 and No.2 of Moldavan WBS satisfy the formulated conditions (Tab. 1). There are no thalwegs on the Ploska catchment and on the runoff plots, there is a thalweg 0.74 km long on the Sukha catchment, but the average thalweg slope of 21.6 ‰ differs slightly from the average slope of the catchment equal to 23.4 ‰.

The Ploska catchment (Fig. 2) is the upper channelless part of the slope of a small right tributary

S	
	S

					Average	Average
Catchment	Period of	Number	Area,	Length,	slope of	slope of
	observations	of years	km ²	m	catchment,	thalweg,
					‰	‰
Ploska, Boguslav FEHB ¹	1970-1987	18	0.085	450	24.7	-
Sukha, Veliko-Anadol WBS ²	1956-1987	31	0.63	740	23.4	21.6
Runoff plots No.1 and No. 2, Moldavan WBS	1963-1982	17	0.0008	40	111	-

¹Boguslavskaya Field Experimental Hydrological Base of Ukrainian Research Hydrometeorological Institute;

²Veliko-Anadol Water-Balance Station, Donetsk region;

³Moldavan Water-Balance Station, Republic of Moldova.

Analysis of materials of the Pridesnyanskaya Runoff Station, the Veliko-Anadol Water-Balance Station, the Boguslav Field Experimental Hydrological Base of the Ukrainian Research Hydrometeorological Institute, located within the territory of Ukraine, and the Moldavian Water-Balance Station located relatively close to the borders of Ukraine on the of the Butenya River (tributary of the Rosava River, the right tributary of the Dnieper River), within the Boguslavskiy district of Kyiv region. Its area is 8.5 ha; the average slope is 24.7‰ (Tab. 1). Maximum slope is 54.4‰. Slope exposition is northern. Soils are dark grey forest podzolized, coarse-dusty light loamy, mostly non-eroded, lightly and medium eroded in the



Fig. 2. Main input, derivatives and resultant digital maps of the Ploska catchment: a - digital elevation model; b - slope steepness map; c - slope longitudinal curvature map; d - upstream elements map (shows a surface flow structure); e - the hydrometeorological factor map; f - soil losses map

lower part only. During the period under consideration, the surface of the Ploska site was fully ploughed and used for cultivation of agricultural crops as one crop rotation field (winter wheat - 47 % years, corn – 28 % years, sugar beet – 14 % years, as well as peas and perennial grasses – 5 % years each).

The size of the raster for all digital maps of the catchment which are used for calculation of the soil losses rate is 90×140 . The size of the cell is 5 m.

catchment of the Veliko-Anadol Water-Balance Station located in Donetsk region, is a left-bank tributary of the Sukha Volnovaha river (the Kalmius river basin). The catchment area is 0.63 km², thalweg length is 0.8 km, the average width of the catchment is 0.57 km, the average slope of the thalweg is 21.6 ‰, average catchment slope is 23.4 ‰. The soil cover of the catchment is homogeneous and is represented by ordinary chernozem, which thickness is 50-75 cm.

Catchment	Average ann W t/	Calculation					
	actual	calculated	error, %				
Ploska, Boguslav FEHB	0.33	0.601 / 0.392	81.8/18.2				
Sukha, Veliko-Anadolskaya WBS	0.16	0.121	-25.0				
Runoff plot No.1, Moldavan WBS	6.24	8.19 ¹	31.3				
Runoff plot No.2, Moldavan WBS	7.6	10.61	39.5				

Table 2. Comparison of calculated and actual average annual rainstorm soil loss by test catchment

¹ calculated using the zonal norm of the hydrometeorological factor of rainstorm soil loss;

² calculated using the average for the period of observations on the catchment of the hydrometeorological factor, calculated according to the nearest weather station data.

The zonal value of the hydrometeorological factor is 0.0033. The main input, derived and resulting digital maps are presented in Fig. 2. The value of the calculated average annual soil losses is 0.60 t/ha/year, while the value of actual soil losses measured in the outlet during a 17-year period is 0.33 t/ha/year (Tab. 2). A characteristic feature is the distinct alternation of soil outwash and soil sedimentation along the lines of surface runoff concentration. In some cells of the raster, the estimated soil outwash reaches 130 t/ha/yr and sedimentation – 280 t/ha/yr.

The Sukha test catchment is an experimental

Brown loam lies below, turning into yellow clay at a depth of 100-150 cm.

During the period under consideration, the catchment area was almost completely used for growing crops – winter wheat, barley, oats (44% of the area), corn (22%), annual and perennial grasses (22%), vegetable crops (9%), on average 3% of the area was occupied by fallow.

In Fig. 3 a screenshot of the PCRaster working window with the result of calculating the rate of storm soil losses within the Sukha catchment in tons per hectare on average per year with superimposed



Fig. 3. Map of the calculated norm of erosion-deposition (t/ha/yr) within the Sukha catchment, Veliko-Anadol Water-Balance Station

contour lines of relief after 2 m is presented. The averaged over area annual soil loss is 0.12 t/ha/year, which is very close to the average annual soil loss determined from observations on the catchment for 31 years (0.16 t/ha/year). Attention is drawn to the significant accumulation of sediments in the thalweg, despite the fact that its slope is quite comparable with the average slope of the catchment.

Runoff plots No.1 and No.2 of the Moldavan Water-Balance Station, which is located in the central part of the Republic of Moldova, 18 km south-east of Chisinau, during the period under consideration, were used for growing crops in crop rotation. In the summer period, plot No.1 was occupied by corn during 47 % of years, by melon crops – 13 % of years, by oats – 7 % of years, by beans – 6 % of years and by fallow-27 % of years ($f_a = 0.68$). Plot No.2 during the most part of the period under consideration in the summer was under fallow (during 67% of years), under corn – 13 %, under oats – 7 %, under leguminous crops – 7 %, under melon crops – 6 % of years. ($f_a=0.85$).

Due to the small size of the plots (40x20 m), the lack of data of detailed relief survey, the calculation of the rate of soil losses for the plots was done using average values of length, slope, soil cover and agricultural background characteristics, and zonal value of the hydrometeorological factor (Svetlitchnyi et al., 2004). The calculated values of the soil loss for the plots exceed the actual ones by 31.3 % and 39.5 % (Tab. 2). For the calculation of soil erosion losses, such accuracy can be considered acceptable. The obtained values of erosion soil losses with sufficient accuracy from a practical point of view characterize the intensity of erosion processes and can be used to assess the erosion hazard of the territory, and to justify the soil protection measures.

But it should be noted that, on the magnitude of the calculation error in this case could have had an impact, on the one hand, the use of averaged over the area values of input variables, and, on the other, using of zonal value of the norm of the hydrometeorological factor of rainstorm soil erosion. Zonal value of the KGM were obtained based on the use of meteorological observation data at the reference meteorological stations of the Steppe and Forest-Steppe zones of Ukraine for 1949-1989, followed by a generalization of the results of calculations for individual meteorological stations within homogeneous regions using the ergodicity hypothesis (Svetlitchnyi, 1995; Svetlitchnyi et al., 2004). It is likely that the perennial mean value of the hydrometeorological factor for the period of observations at the test catchment differs from the zonal norm.

So, the perennial mean value of the hydrometeo-

rological factor for the catchment, calculated for the period 1970-1987 using data of the nearest meteorological station Boguslav amounted to 0.0019, while the zonal value is equal to 0.0033. Accordingly, the calculated average value of soil loss using this perennial mean value of the hydrometeorological factor has been obtained equal to 0.39 t/ha/yr, which is much closer to the actual (0.33 t/ha/yr) than using the zonal K_{HM} value.

Comparison of calculated and actual values of mean annual erosion losses of soil for all test catchments is presented in Table 2. Given the large amplitude of the soil loss rate, to assess the accuracy of the model it is advisable to use not the mean square error of calculations, but more informative criterions, taking into account the increased variability of the annual soil losses. For four test catchments, for example, the minimum and maximum averaged over the area values of the annual rainstorm soil losses (0.16 and 7.60 t/ha/yr) differ by almost 50 times.

To assess the quality of the developed model we use the Nash-Sutcliffe (NS) efficiency criterion (Nash & Sutcliffe, 1970), which according to (Vinogradov & Nikiforovskiy, 2015) is currently the most popular criterion among specialists in mathematical modeling. The use of this criterion, in particular, is recommended by the American Association of Civil Engineers to assess the viability of runoff models. It is actively used to estimate mathematical models of water soil erosion (Van Rompaey et al., 2003; Wang et al., 2018 etc.). In the general case, the quality of the model is considered satisfactory for NS> 0.5, for 0.65 <NS \leq 0.75 - as good and at 0.75 <NS \leq 1.00 - as very good.

The NS criterion is calculated according to the formula

$$NS = 1 - \frac{\sum_{i=1}^{n} (W_{i \, act} - W_{i \, calc})^{2}}{\sum_{i=1}^{n} (W_{i \, act} - \overline{W}_{i \, act})^{2}},$$
(7)

where W_{iact} and W_{icalc} are the actual (observed) and the calculated values of the variable;

 $W_{i act}$ is average actual value of the variable; *n* is a number of values of the variable.

The value of the Nash-Sutcliff criterion for the developed model based on the results of the calculation for the four test catchments is 0.72, which makes it possible to evaluate the quality of the model as good.

Conclusions. 1. The GIS-implemented physicalstatistical model of erosion-sedimentation takes into account the main features of rainstorm surface runoff and soil erosion on the slopes of the Steppe and Forest-Steppe zones of Ukraine, including slopes with a complex shape of a longitudinal and transverse profile. The conditions limiting the account of influence of the transverse concentration of surface runoff by the model are the presence of a detailed hydrologically correct digital elevation model and the raster cell size of not more than 10 m.

2. Practically the only source of data that allows assessment of the adequacy of soil erosion models in the natural-economic conditions of Ukraine remains the observational materials at the Runoff and Water-Balance stations of the Hydrometeorological Service, which were collected in the 1960–1980s..

3 The performed verification calculations of soil erosion losses for two runoff plots of the Moldavan Water-Balance Station and two small catchments of the Boguslav Field Experimental Hydrological Base and Veliko-Anadol Water-Balance Station, with observation periods of 17-31 years, made it possible based on the Nash-Sutcliff criterion to assess the quality of the developed model as good.

References

- Ascough II, J.C, Baffaut, C, Nearing, M.A, Liu, B.Y., 1997. The WEPP watershed model: I. Hydrology and erosion. Transactions of the ASAE, 40(4), 921– 933.
- Chorny, S. G., 1996. Skhylovi zroshuvani ahrolandshafty: eroziia, hruntoutvorennia, ratsionalne vykorystannia [Slope irrigated agro-landscapes: erosion, soil formation, rational use]. Borisphen, Kherson, (in Ukrainian)
- De Roo, A.P.J, Wesseling, C.G, Ritsema, C.J., 1996. LISEM: a single event physically-based hydrologic and soil erosion model for drainage basins. I: Theory, input and output. Hydrological Processes, 10, 1107–1117.
- Kanash, O., Osypchuk, S., 2003. Erosiia gruntiv Ukrainy: suchasnyi stan, aspekty raionuvannia, tendentsiia bahatorichnykh zmin [Soil Erosion in Ukraine: the present state, aspects of zoning, the tendency of many years of change]. Genesis, geography and ecology of soil. Publishing Center of Ivan Franko National University of Lviv, Lviv, 158-163 (in Ukrainian)
- Koval'chuk, I.P., Yevsyukov, T.O., Mkrtchian, O.
 S., Lobans'ka, N. I., 2009. Heoprostorove modeliuvannia potentsialu rozvytku dehradatsiinykh protsesiv na ornykh zemliakh [Geospatial modeling of the potential of degradation processes on arable land]. Land system and cadaster, 4, 72-82 (in Ukrainian)
- Lavrovskiy, A.B., Igumentcev, A.F., Anisimov, S.V., Shchegoleva, L.G., 1987. K voprosu postroeniya

modeli stoka i smyva pochvy pri livnevoi erozii [On the question of constructing a model of flow and soil losses during shower erosion]. Proc. All-Union conf. "The regularities of manifestation of erosion and channel processes in various natural conditions". Publishing Mosk. Univ., Moscow, 89-90 (in Russian)

- Morgan, R.P.C, Quinton, J.N., Smith, R.E., Govers, G., Poesen, J.W.A., Auerswald, K., Chisci, G., Torri, D., Styczen, M.E., 1998. The European soil erosion model (EUROSEM): a dynamic approach for predicting sediment transport from fields and small catchments. Earth Surface Processes and Landforms, 23, 527-544.
- Natsionalna dopovid pro stan rodiuchosti gruntiv Ukrainy [National report on soil fertility in Ukraine], 2010. Baliuk, S.A., Medvedev, V.V., Tarariko, O.G., Grekov O.V., Balaev, A.D. (eds). VIK PRINT, Kyiv (in Ukrainian).
- Nash, J.E., Sutcliffe, J.V., 1970. River flow forecasting through conceptual models part I — A discussion of principles. Journ. Hydrology, 10(3), 282–290.
- PCRaster: Software for Environmental Modeling, 2018. Retrieved from: http://pcraster.geo.uu.nl/ downloads/latest-release/.
- Pravila po opredeleniju maksimal'nyh rashodov vody dozhdevyh pavodkov s malyh vodosborov pri proekti-rovanii protivojerozionnyh gidrotehnicheskih sooruzhenij na ravninnoj territorii USSR. VSN 10.15 USSR 1-87 [Rules for determining the maximum flow of rainwater floods from small catchments when designing anti-erosion hydrau-lic structures in the lowland territory of the USSR. VSN 10.15 USSR 1-87], 1987. Kyiv (in Russian).
- Pyatkova, A.V., 2008. Osobennosty modelirovaniya vodnoy erosiyi s uchetom prostranstvennoy izmenchivosty eye factorov [Features of soil water erosion modeling taking into account spatial changeability of its factors], Mete-orology, Climatology and Hydrology, 50,437-442 (in Russian).
- Piatkova, A.V., 2013. Urakhuvannia struktury skhylovoho stikannia pry prostorovomu modeliuvanni zlyvovoho zmyvu gruntu [Taking into account the surfact flowing structure on the space modeling of rainfall soil losses]. Gerald of the Odessa National University. Series:Geography & Geology, 18, 82-87 (in Ukrainian).
- Renard, K.G., Foster, G.R., Weesies, G.A., McCook, D.K., Yoder, D.C., 1993. Predicting soil erosion by water: a guide to conservation planning with the Revised Universal Soil Loss Equation (RUSLE), Agriculture Handbook #703, U.S. Department of Agriculture, Washington.
- Schindewolf, M.; Schmidt, J., 2012. Parameterization of the EROSION 2D/3D soil erosion model using a small-scale rainfall simulator and upstream runoff simulation. Catena, 91, 47–55.

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- Shvebs, H.I., 1974. Formirovaniye vodnoy erozii. stoka nanosov i ikh otsenka [Formation of water erosion, sediment yield and their assessment]. Hydrometeoizdat, Leningrad (in Russian).
- Shvebs, H.I., 1981. Teoreticheskiye osnovy eroziovedeniya [Theoretical foundations of erosion]. Vyshcha shkola, Ky-iv-Odesa (in Russian).
- Smith, R.E, Goodrich, D.C, Quinton, J.N., 1995. Dynamic, distributed simulation of watershed erosion: The KINEROS2 and EUROSEM models. Journal of Soil and Water Conservation, 50, 517–520.
- Spravochnik po pochvozashhitnomu zemledeliju, 1990 [Handbook of Soil Protective Agriculture]. I. N. Bezruchko, L.Ya. Milchevskaya (eds). Urozhay, Kiev (in Russian).
- Sribnyi, I.A., 1977. Srednegodovoy stok vody i smyv pochvy so sklonov [The average annual water flow and soil out-wash from the slopes]. Water construction on small rivers. Budivelnik, Kiev, 145-147 (in Russian).
- Sribnyi, I.A., Vergunov, V.A., 1993. Vyznachennia zmyvu hruntu zi skhyliv [Determination of the soil losses from the slopes]. Herald of agrarian science, 7, 42-46 (in Ukrainian).
- Svetlitchnyi, A.A., 1995. Prostranstvenno-vremennaya otsenka livnevoy erozii pochv [Spatial-temporal assessment of soil erosion], Dep. SSTB of Ukraine, №1911-Uk95 (in Russian).

- Svetlitchnyi, A.A., 1999. Printsipyi sovershenstvovaniya empiricheskih modeley smyiva pochvyi [The principals of improving empirical models of soil erosion]. Eurasian Soil Science, 32(8), 917-923.
- Svitlychnyi О. O., Ivanova V., 2003. Α. modelyuvannya Pryncypy prostorovogo gidrometeorologichnyh umov zlyvovogo zmyvu gruntu [Principles of spatial modelling of hydrometeorological conditions of soil storm wash off]. Gerald of the Odessa National University. Series: Geography & Geology, 8, 77-82 (in Ukrainian).
- Svetlitchnyi, A.A., Shvebs, H.I., Cherny, S.G., 2004. Eroziovedenie: teoreticheskie i prikladnyie aspektyi [Soil erosion science: theoretical and applied aspects]. VTD "University Book", Sumy (in Russian).
- Svetlitchnyi, A.A., Plotnitskiy, S.V., Stepovaya, O.Y., 2003. Spatial distribution of soil moisture convent within catchments and its modeling on the basis of topographic data. Journal of Hydrology, 277, 50-60.
- Vinogradov A., Nikiforovskiy A., 2015. Analiz sootvetstvija kriteriev kachestva modelirovanija processov formirovani-ja stoka malyh rek [Quality conformity analysis of runoff formation modeling criteria for small rivers]. Herald of Moscow State University. Series: Nature Sciences, 2, 33-38.